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A PROBLEM IN MECHANICAL FLIGHT.

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In 1883 Lord Rayleigh pointed out* that whenever a bird pursues its course for some time without working its wings, we must conclude "either (1) that the course is not horizontal; (2) that the wind is not horizontal; or (3) that the wind is not uniform. It is probable that the truth is usually represented by (1) or (2);" and adds "but the question I wish to raise is whether the cause suggested by (3) may not sometimes come into operation."

It will be noticed that case (1) of this analysis is not adequately defined, since it is not expressed like the others in terms of the wind. A further examination of the article discloses that this case is that in which the bird by ascending and descending takes advantage of *winds of different velocity at different altitudes*. Lord Rayleigh expressed his distrust of the adequacy of either (1) or (2) to account for the observed facts, but did not investigate the efficiency of (3). The matter has rested at this point until a very recent period. In a memoir recently published by Professor S. P. Langley,† it is shown that the condition represented by (3) always exists; that the wind is never a homogeneous current but consists of a continued series of rapid pulsations varying indefinitely in amplitude and period. The periods are measured by seconds rather than by minutes, and the amplitudes increase with the wind velocity. Having established the fact that the wind is not uniform, the author then gives a popular explanation of the mechanical principles which enable the bird to utilize such pulsations in maintaining its flight without working its wings, or expending energy. This may be accomplished by a succession of ascents and descents; the ascents being made during the wind gusts, and the descents during the lulls.

Manifestly, however, in order to show that this third case of Lord Rayleigh's analysis is the one *actually employed* by the bird in soaring, it is necessary to show that the pulsations in addition to being qualitatively applicable are also quantitatively sufficient. The bird must rise as high in his ascents as he falls in his descents, and this equality of altitude must be shown to be possible for any assigned wind pulsations in which soaring occurs.

With this application in view, the present paper seeks to determine the course in the air of a free heavy plane subjected to definite wind pulsations.

* *Nature*, April 5, 1883.

† The internal work of the wind. By S. P. Langley. Smithsonian Contributions to Knowledge. Washington, 1893.

Since the actual pulsations of the wind exhibited by the anemometer records are in every case too complex to be treated analytically at this stage of the investigation, I shall follow a suggestion made for this purpose by Professor Langley, and consider that a homogeneous wind blows for a very short period at a uniform rate, then that there is an equal period of calm, and so on alternately. The body immersed in the wind is supposed to be a material plane whose front edge (transverse to the wind) is six times its width, and whose surface is n square feet to the pound (of its weight). For numerical computation the periods of wind and of calm will first be assumed to be five seconds each. Let us take up the investigation of the motion at the beginning of a period of calm, when we will suppose the plane to be momentarily at rest at a given height in the air, and to be capable of changing its angle without expenditure of energy. Since the initial angle which the plane assumes is not conditioned, we will suppose it to be 60° below the horizontal, which is approximately that sometimes taken by birds in the beginning of a rapid descent, and that under the force of gravity the plane glides down the air in the curve of quickest descent (the inverted cycloid) until its course becomes horizontal at the vertex.

If s = any distance measured along the curve, and

a = radius of the generating circle,

the equation of the cycloid, in which the lowest point of the curve is taken as the origin and the axis of y is vertical, is

$$s^2 = 8ay, \text{ and } \frac{ds}{dy} = \sqrt{\frac{2a}{y}}.$$

Since the initial angle of the plane with the horizon is 60° , we have

$$\frac{dy_1}{ds} = \sin 60^\circ = \frac{1}{2} \sqrt{3},$$

y_1 being the ordinate of the initial position ;

$$\therefore \frac{ds}{dy_1} = \frac{2}{\sqrt{3}} = \sqrt{\frac{2a}{y_1}}, \text{ or } \frac{2a}{y_1} = \frac{4}{3},$$

an equation between a , the radius of the generating circle, and the distance of the fall y_1 . Let the fall of the plane be 36 feet ; we then have a cycloid in which $a = 24$ feet.

The time of fall from any point on an inverted cycloid to the vertex is

$$t = \pi \sqrt{\frac{a}{g}}.*$$

* Tait and Steele: Dynamics of a particle, p. 173.

Whence, in this case if the air reacted like a smooth solid on the gliding plane, and the plane moved tangent to itself, we should have

$$t = \pi \sqrt{\frac{24}{g}} = 2.72 \text{ secs.},$$

and the velocity of the plane, which is given by the expression

$$\left[\frac{ds}{dt} \right]^2 = \frac{g}{4a} (s_1^2 - s^2)$$

would become at the vertex of the cycloid

$$\left[\frac{ds}{dt} \right]^2 = \frac{g}{4a} s_1^2, \quad \text{or} \quad V = \sqrt{2gy_1} = 48 \text{ feet per second.}$$

But since the air does not react like a solid, the plane in order to take the assumed cycloidal path cannot move forward in its own plane, but must make a small angle α with the path, such that the sustaining component of the air pressure shall be just sufficient to balance the force of gravity to the required extent. This resistance of the air will cause a lengthening of the time and a diminution of the velocity with which the plane reaches the vertex of the cycloid.

The equation of motion is

$$\frac{d^2s}{dt^2} = -g \cdot \frac{s}{4a} + kn g \left[\frac{ds}{dt} \right]^2 F(a) \sin \alpha,$$

in which s is measured from the vertex. The first term of the second member is the acceleration due to gravity, and the second term is the retardation due to the resistance of the air acting on the plane at the angle α . k = the constant of air pressure = 0.00166 lbs. per sq. foot for a velocity of 1 foot per second.

Let $n = 2.3$, which is a ratio of surface (in sq. feet) to weight (in lbs.) found in certain species of soaring birds. $F(a)$ = the ratio of pressure on an inclined plane to the pressure on a normal plane, determined experimentally for rectangles of various shapes in *Experiments in Aerodynamics* by S. P. Langley.

The angle α probably varies within small limits at different parts of the cycloidal path, but for a first approximation in order to be able to integrate, it will here be taken as constant.

Let

$$kn g F(a) \sin \alpha = c.$$

Then

$$\frac{d^2s}{dt^2} - c \left[\frac{ds}{dt} \right]^2 = -g \frac{s}{4a};$$

whence, multiplying through by $e^{-2cs}ds$, we obtain

$$2 \frac{d^2s}{dt^2} \left[\frac{ds}{dt} \right] e^{-2cs} - 2 c e^{-2cs} \left[\frac{ds}{dt} \right]^2 ds = - \frac{g}{2a} s e^{-2cs} ds .$$

The first member is now a complete differential of which the integral is $\left[\frac{ds}{dt} \right]^2 e^{-2cs}$;

$$\therefore \left[\frac{ds}{dt} \right]^2 e^{-2cs} = - \frac{g}{2a} \int s e^{-2cs} ds + \text{const.}$$

Integrating the second member by parts,

$$\left[\frac{ds}{dt} \right]^2 = \frac{g}{2a} \left[\frac{1 + 2cs}{4c^2} \right] + e^{2cs} \cdot \text{const.}$$

To determine the constant, we have $s = s_1$ when $\frac{ds}{dt} = 0$; hence

$$\text{const.} = - \frac{g}{2a} \cdot \frac{1 + 2cs_1}{4c^2} \cdot e^{-2cs_1} ,$$

and

$$\left[\frac{ds}{dt} \right]^2 = \frac{g}{2a} \left[\frac{1 + 2cs}{4c^2} - \left[\frac{1 + 2cs_1}{4c^2} \right] e^{2c(s-s_1)} \right] .$$

By putting $s = 0$, the velocity at the vertex of the cycloid is given by the equation

$$\left[\frac{ds}{dt} \right]_{s=0}^2 = \frac{g}{2a} \left[\frac{1}{4c^2} - \left[\frac{1 + 2cs_1}{4c^2} \right] e^{-2cs_1} \right]$$

In order to evaluate this velocity we must determine the numerical values of the constants.

From the equation of the cycloid,

$$s_1 = \sqrt{8ay_1} = 48 \sqrt{3} = 83.1 .$$

c was substituted for the expression $kng F(a) \sin a$ in which we must assign a value for the angle a .

From the numerical results obtained in later portions of this analysis, 2° is here adopted as an approximate mean value for a , and from *Experiments in Aerodynamics*, p. 62, $F(a) = F(2^\circ) = 0.15$. With these values and others already given,

$$c = 0.00166 (2.3) (32\frac{1}{8}) (0.15) (0.0349) = 0.000643 ,$$

$$2cs_1 = 0.10687 , \quad e^{2cs_1} = 1.113 , \quad \frac{ds}{dt} = 47.2 \text{ feet per sec.}$$

Comparing this result with that above given for a body sliding down a solid frictionless cycloid, we find that the velocity at the vertex is 0.8 foot per second smaller, or 47.2 instead of 48.0 feet per second.

The time of reaching the vertex is given by the integral

$$t = \int_{s=s_1}^{s=0} \frac{ds}{\sqrt{\frac{g}{2a} \left[\frac{1+2cs}{4c^2} - \left(\frac{1+2cs_1}{4c^2} \right) e^{2c(s-s_1)} \right]}}.$$

This expression is not directly integrable, but its value can be computed approximately by the method of quadratures. A computation by this method of the time between $s = 0$ and $s_1 = 65$ (the portion of the path for which the retardation is appreciable), gives .03 second more than the time given by the simpler formula; making the time for describing the whole path 2.75 seconds instead of 2.72 seconds.

There still remain then 2.25 seconds of the assigned period of calm. We may assume that the plane, after reaching the vertex of the cycloid with a horizontal velocity v , at once assumes an upward inclination of 7° , the front edge being elevated.

The vertical component of air pressure on the plane due to its own motion will be*

$$kA V^2 F(a) \cos 7^\circ,$$

in which k = the constant of air pressure = .00166 lbs. per sq. foot;

$A = nW$ = area of surface in sq. feet;

V = relative velocity of air and plane in feet per second;

a = angle of inclination between plane and path of advance;

$F(a)$ = ratio of pressure on an inclined plane to the pressure on a normal plane, determined experimentally for rectangles of various shapes in *Experiments in Aerodynamics*.

At the vertex of the cycloid the path of the plane is horizontal, and hence initially

$$a = 7^\circ,$$

$V = 47.2$ feet per second, as just determined, and

$A = 2.3 W$ (weight of plane), by hypothesis.

For $a = 7^\circ$, $F(a)$, for a plane whose length is 6 times its width, equals 0.36. (*Experiments in Aerodynamics*, Diagram, p. 62.)

Substituting these values in the expression just given, we find that the vertical upward component of pressure on the plane is 3.15 W , or 3.15 times

* See *Experiments in Aerodynamics*, p. 60.

the weight of the plane, and therefore materially exceeds the force of gravity. The motion of the plane will consequently have a vertical upward component.

As the plane begins to rise, the angle between the plane and the wind of advance diminishes, and hence the vertical component of pressure diminishes. Uniform motion will be attained when the vertical component of pressure equals the weight.

For this condition, we shall have the equation

$$W = 0.00166 \times 2.3 \times W \times 47.22 \times F(a) \cos 7^\circ$$

to determine a .

$$1 = 3.70 \times 2.3 \times F(a) \cos 7^\circ,$$

$$F(a) = .118,$$

$$a = 1\frac{1}{2}^\circ. \quad (\textit{Experiments in Aerodynamics}, \text{p. 62})$$

This result means that the plane will take up a path making an angle of $5\frac{1}{2}$ degrees with the horizontal, while its own angle is 7° . This cannot, in fact, be done instantaneously, but the time required is so short that for our purpose we may consider this to be the condition at the outset of the motion. The initial velocity of 47.2 feet per second will now be subject to diminution from the resistance of the air due to the angle of $1\frac{1}{2}^\circ$ between the plane and the wind of advance. As the velocity diminishes, the angle a must increase in order that the equation between weight and vertical pressure shall be preserved. The increase of a in turn brings about an increase in the horizontal resistance, and hence it follows that the velocity of the plane decreases at an increasing rate.

The differential equations of motion are

$$\frac{W}{g} \cdot \frac{d^2x}{dt^2} = -kn W V^2 F(a) \sin 7^\circ, \quad (1)$$

$$\frac{W}{g} \cdot \frac{d^2y}{dt^2} = +kn W V^2 F(a) \cos 7^\circ - W, \quad (2)$$

in which V and a are variables.

In order to integrate, let us divide the period into a series of such short intervals that for each of these a may be assumed without appreciable error to be constant.

For the first interval, let $a = 1^\circ 30'$. Then the angle of path with the horizon will be $5^\circ 30'$, and we shall have

$$V = \frac{dx}{dt} \sec(7^\circ - a) = \frac{dx}{dt} \sec 5^\circ 30',$$

and

$$\frac{d^2x}{dt^2} = -kngF(a) \sin 7^\circ \left[\frac{dx}{dt} \right]^2 \sec^2 5^\circ 30'.$$

To integrate this, put

$$c' = kn g F(a) \sin 7^\circ \sec^2 5^\circ 30', \text{ and } w = \frac{dx}{dt};$$

then

$$\frac{dw}{dt} = -c' w^2, \text{ or } \frac{1}{w^2} \frac{dw}{dt} = -c';$$

whence, integrating,

$$\frac{1}{w} = c't + \text{const.}$$

Substituting the value of w ,

$$\frac{1}{\frac{dx}{dt}} = c't + \text{const.}$$

When $t = 0$, $\frac{dx}{dt} = V_0 \cos(7^\circ - a)$; whence, $\text{const.} = \frac{1}{V_0 \cos(7^\circ - a)}$, and we have

$$\frac{dx}{dt} = \frac{V_0 \cos(7^\circ - a)}{1 + c't V_0 \cos(7^\circ - a)}. \quad (3)$$

Integrating again, we have

$$x = \frac{1}{c'} \log \left[c't + \frac{1}{V_0 \cos(7^\circ - a)} \right] + \text{const.}$$

When $t = 0$, $x = 0$; whence $\text{const.} = -\frac{1}{c'} \log \frac{1}{V_0 \cos(7^\circ - a)}$.

$$\therefore x = \frac{1}{c'} \log \left[\frac{c't + \frac{1}{V_0 \cos(7^\circ - a)}}{\frac{1}{V_0 \cos(7^\circ - a)}} \right] = \frac{1}{c'} \log [1 + c't V_0 \cos(7^\circ - a)]. \quad (4)$$

Substituting the assigned numerical values for the quantities in the equation for c' , we have

$$c' = .00166 \times 2.3 \times 32.18 \times F(1\frac{1}{2}^\circ) \sin 7^\circ \cdot \sec^2 5\frac{1}{2}^\circ.$$

As we have already seen, $F(1\frac{1}{2}^\circ) = 0.118$; hence,

$$c' = 0.00177;$$

$$\frac{dx}{dt} = \frac{47.2 \cos 5\frac{1}{2}^\circ}{1 + .0835 t \cos 5\frac{1}{2}^\circ}, \quad V = \frac{47.2}{1 + .0835 t \cos 5\frac{1}{2}^\circ}.$$

The period for which the integration is to be made is 2.25 seconds, and this interval is so short that we may evaluate the equations for the whole of it without making a material error. Placing therefore $t = 2.25$, we have

$$V = 40 \text{ feet per second,}$$

$$x = 97 \text{ feet.}$$

The altitude gained will be $97 \tan 5\frac{1}{2}^\circ = 9.3$ feet.

At the beginning of the second interval of 5 seconds, the wind, by hypothesis, begins to blow with a uniform velocity. For the purposes of the present example, let this velocity be 36 feet per second. The relative velocity of wind and plane will now be the geometrical resultant of their respective velocities. Let the plane maintain its constant angle of 7° with the horizon. The vertical upward component of air pressure will again give a vertical component to the plane's motion, and the plane will begin to ascend until the angle of path is such that the vertical upward component of pressure is equal to the weight. For a condition of uniform motion we shall have, as before, the equation

$$W = kn W V^2 F(\alpha) \cos 7^\circ,$$

in which, now,

V is the relative velocity of wind and plane ;

α is the angle between the plane and the direction of V .

It will be seen that the preceding equation for evaluating $F(\alpha)$ contains the relative velocity V , which is itself a function of α . But since a change in α makes only a very small change in V , we may compute V by assuming an approximate value of α , and then compute $F(\alpha)$ from the equation, in order to obtain a more accurate value.

Assuming $\alpha = 1^\circ$, we obtain by a solution of the triangle ABC , in which AB represents the horizontal velocity of the wind, CB the velocity of the plane, and AC the relative velocity V ,

$$V = 75.6 \text{ feet per second.}$$

Let this initial value of V be designated V_0 . We have then,

$$F(\alpha) = \frac{1}{kn V_0^2 \cos 7^\circ} = 0.046 ;$$

$$\alpha = \frac{1}{2}^\circ. \quad (\text{Experiments in Aerodynamics, p. 62.})$$

If we substitute this value in the differential equation (1) already given for $\frac{d^2x}{dt^2}$, and notice that now $\frac{dx}{dt}$ denotes *relative* and not *absolute* velocity,

we have

$$\frac{d^2x}{dt^2} = -g \frac{V^2}{V_0^2} \tan 7^\circ = -g \frac{\tan 7^\circ}{V_0^2} \cdot \left[\frac{dx}{dt} \right]^2 \sec^2 (7^\circ - \alpha)$$

Integrating as before, putting

$$g \frac{\tan 7^\circ}{V_0^2} \sec^2 (7^\circ - \alpha) = c' = 0.000695,$$

we have

$$\frac{dx}{dt} = \frac{V_0 \cos (7^\circ - \alpha)}{1 + c't V_0 \cos (7^\circ - \alpha)},$$

and

$$x = \frac{1}{c'} \log [1 + c't V_0 \cos (7^\circ - \alpha)].$$

Since the integration has been made on the condition that t be taken as a short interval for which α is sensibly constant, we will divide the whole period of 5 seconds into two equal parts, computing the position of the plane at the end of $2\frac{1}{2}$ seconds, and with its velocity at this position, make a second computation for the remaining $2\frac{1}{2}$ seconds.

$$\text{With } t = 2\frac{1}{2}, \quad \frac{dx}{dt} = 66.9;$$

$$x = 177 \text{ feet.}$$

The relative velocity at the end of the interval is

$$V = \frac{dx}{dt} \sec 6\frac{1}{2}^\circ = 67.3.$$

The *absolute* horizontal distance traversed by the plane $= 177 - 2\frac{1}{2}(36) = 87$ feet.

The angle of the path of plane with the horizon becomes known after computing the value of the angle C in the triangle ABC . The known parts are now $A = 6^\circ 30'$, $AB = 36$, and AC , which is given a mean value between the value $V_0 = 75.6$ at the beginning of the period, and $V = 67.3$ at the end. α being taken as $0^\circ 30'$, the angle of the path with the horizon is found to be 13° . The ascent made by the plane in two and a half seconds will be

$$y = 87 \tan 13^\circ = 20.1.$$

Taking now the value $V = 67.3$ as the initial relative velocity V_0 , for the remaining two and a half seconds, and repeating the computation we have the following result:

$$F(\alpha) = 0.058; \quad \alpha = 0^\circ 40'; \quad c' = 0.00088$$

$$\frac{dx}{dt} = 58.5; \quad x = 157 \text{ feet}; \quad V = 58.9.$$

The absolute horizontal distance travelled $= 157 - 2.5 \times (36) = 67$ feet. The mean angle of path computed as before, is found to be $14^\circ 36'$, and the ascent made by the plane is $67 \tan 14^\circ 36' = 17.5$ feet.

The total height gained by the plane after reaching its lowest point at the vertex of the cycloid, is the sum of the separate heights gained in the three intervals for which we have made the computation, namely, 9.3, 20.1 and 17.5 feet, making 46.9 feet in all. (This result is slightly greater than would be given by a perfect integration, since α is not absolutely constant for the intervals used.) Thus, without any internal source of energy, during the 10 seconds of alternate calm and wind, the plane has in the first 2.75 seconds made a descent of 36 feet, and in the remaining 7.25 seconds has risen 46.9 feet, travelling at the same time horizontally a distance of 251 feet, of which 154 feet is made against the wind. In addition, the relative velocity of the plane and the wind (58.9 feet per second) at the end of this period is sufficient, if the wind continue with the same velocity, to yield a considerable further ascent before the vertical component of pressure is reduced to such an extent that it no longer exceeds the weight of the plane under the constant angle of inclination adopted.

Evidently, the trajectory described by the plane during the first period of ten seconds, can be repeated indefinitely so long as the wind possesses the assigned pulsations. The problem stated at the outset of the paper has therefore been solved for a single case in which the wind pulsations are of a simple character. From the results obtained in the course of the solution, a number of interesting generalizations are immediately deducible as to the effect of a change in the value of the constants which enter the formulæ. In every case a change in the numerical values affects the soaring plane favorably or unfavorably, and for each one limiting values can be found beyond which soaring will be impossible, all the other conditions remaining unchanged.

Up to a certain limit soaring is facilitated by a decrease in the value of n , the number of square feet of sustaining surface to a pound of weight in the soaring plane or bird. For a given area of wing, the heavier bird possesses the greater momentum, and sails with greater steadiness amid all the varying gusts of the pulsating wind. With respect to the wind, soaring is facilitated by an increase in the amplitude, a , of the wind pulsations and by a decrease in the interval, t , of their recurrence. It may be observed also that in general an advantage accrues when the period of lull is shorter than the period of gust. The relative numerical values of n , a , and t are the principal factors affecting the possibility of soaring in any specific case. It should also be remarked here that certain forms of curved surface, like the bird's wing, are so much more advantageous than planes, that soaring is possible for

them in wind pulsations which would not sustain the simple plane ; but the pressure constants applicable to such surfaces have not yet been determined.

In the example here solved, the wind-lulls have been assumed as actual calms, and the plane has moved in a straight forward path. This condition seldom occurs in nature, for even in intervals of relative calm there is generally some wind, and for a straightforward path this wind tends to strike the descending body on the upper side of its supporting surface. I have made the suggestion that the soaring bird avoids such a position when possible, and that this is a reason for the circling movement so generally adopted in soaring flight.